

Constraints on a New Post-General Relativity Cosmological Parameter

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A new cosmological variable is introduced which characterizes the degree of departure from Einstein’s General Relativity (GR) with a cosmological constant. The new parameter, ϖ , is the cosmological analog of γ , the parametrized post-Newtonian variable which measures the amount of spacetime curvature per unit mass. In the cosmological context, ϖ measures the difference between the Newtonian and longitudinal potentials in response to the same matter sources, as occurs in certain scalar-tensor theories of gravity. Equivalently, ϖ measures the scalar shear fluctuation in a dark energy component. In the context of a “vanilla” Λ CDM background cosmology, a non-zero ϖ signals a departure from GR or a fluctuating cosmological constant. Using a phenomenological model for the time evolution $\varpi = \varpi_0 \rho_{\text{DE}}/\rho_{\text{M}}$ which depends on the ratio of energy density in the cosmological constant to the matter density at each epoch, it is shown that the observed cosmic microwave background (CMB) temperature anisotropies limit the overall normalization constant to be $-0.4 < \varpi_0 < 0.1$ at the 95% confidence level. Existing measurements of the cross-correlations of the CMB with large-scale structure further limit $\varpi_0 > -0.2$ at the 95% CL. In the future, integrated Sachs-Wolfe and weak lensing measurements can more tightly constrain ϖ_0 , providing a valuable clue to the nature of dark energy and the validity of GR.

I. INTRODUCTION

There is a gap in our understanding of the cosmos: we do not understand the cause of the accelerated cosmic expansion and the source of most of the energy of the universe. In the context of Einstein’s General Relativity (GR), the evidence suggests the existence of some form of dark energy, such as a cosmological constant or quintessence. (For a review see Refs. [1, 2, 3] and references therein.) Alternatively, novel gravitational phenomena on the largest, cosmological scales may be responsible for the observations (*e.g.* Refs. [4, 5, 6, 7, 8, 9, 10]). One of the outstanding challenges to physics and astronomy is to discover the answer to this problem. Our goal is to reformulate the degeneracy between dark energy and novel gravitational phenomena into a simple formalism. In doing so, we aim to extend the parametrized post-Newtonian formalism (see also Refs. [11, 12, 13, 14, 15]), which has served so well to test GR within the solar system, to cosmological scales.

The plan of the paper is as follows. In section II we motivate and introduce the new cosmological parameter, ϖ , making a connection with the solar system PPN parameters. We show that this parameter may characterize new gravitational physics, but is also degenerate with shear perturbations which we attribute to the dark energy. In section III we propose a model for the parameter, ϖ , detail our procedure for the computation of cosmological observables, and present a sample of the consequences of

$\varpi \neq 0$. Our procedure is not unique but has practical advantages. In sections IV and V we present constraints on ϖ and forecasts using the cosmic microwave background (CMB), cross-correlations with large-scale structure, and weak lensing. A summary with conclusions is given in section VI. We include an appendix explaining how dark energy with $w = -1$ can support fluctuations.

II. COSMIC PPN

General Relativity (GR) tells us precisely how mass and energy curve spacetime. In other theories of gravity, the relationship between between matter and curvature is different. By presuming GR and looking for contradictions we can hope to test the consistency of GR. Post-Newtonian variables are used to quantify the behavior of gravity and departures from General Relativity. (See Refs. [16, 17] for a thorough review.) The Eddington-Robertson-Schiff metric

$$ds^2 = - \left[1 - 2\alpha \frac{GM}{r} + 2(\beta - \alpha\gamma) \left(\frac{GM}{r} \right)^2 + \dots \right] dt^2 + \left[1 + 2\gamma \frac{GM}{r} + \dots \right] dr^2 + r^2 d\Omega^2 \quad (1)$$

can be used to parametrize the way a point mass, M , curves space. The Newtonian potential sets Newton’s constant so that it is convention to set $\alpha = 1$ at the present day. A more detailed metric and system of

parameters is possible, allowing for the motion of the source, though is unnecessary at this stage. The parameter β measures the amount of non-linearity in gravitational superposition, and γ measures how much curvature is produced per unit rest mass. In GR, $\beta = \gamma = 1$. The most recent limits are $\beta - 1 = 1.2(\pm 1.1) \times 10^{-4}$ from lunar laser ranging [18], and $\gamma - 1 = 2.1(\pm 2.3) \times 10^{-5}$ measured from the time-delay of signals from the Cassini spacecraft [19]. Einstein's General Relativity is in excellent agreement with observations within the solar system.

These measurements place very tight constraints on extensions of GR. However, it is possible to imagine that the underlying theory of gravitation resembles GR on solar system scales, but departs on larger, cosmological scales. The post-Newtonian variable γ is very close to 1 within the solar system, but may depart on larger scales or vary with cosmic age. With the notion in mind that dark energy phenomena may be a result of a departure from GR, it is tempting to speculate that a departure from GR hinges on the local ratio of dark energy to the matter density. Using standard cosmological numbers, this ratio is

$$\langle \rho_{DE} \rangle / \langle \rho_M \rangle = \Omega_{DE} \frac{3H_0^2}{8\pi G} / (0.1 M_\odot / \text{pc}^3) \approx 10^{-6}. \quad (2)$$

Consequently, one may speculate that an improvement in solar system tests by an order of magnitude may reveal faults in GR. Alternatively, it may be easier to search for such faults using cosmological phenomena, such as the CMB, the growth of large-scale structure [9, 10], and gravitational lensing [20, 21, 22].

To adapt the post-Newtonian formalism to cosmology, we replace the Eddington-Robertson-Schiff metric with the perturbed FRW metric

$$ds^2 = a^2(\tau) [-(1 + 2\psi)d\tau^2 + (1 - 2\phi)d\vec{x}^2]. \quad (3)$$

Hereafter we employ the notation and conventions of Ma & Bertschinger (MB) [23]. It is convenient to work in the conformal-Newtonian gauge if we wish to maintain the simple connection to the Newtonian potential as appears in the Poisson equation. If the gravitating source is a point mass, then we mean to identify $\psi = -\alpha GM/r$ and $\phi = -\gamma GM/r$, so that measurements of the Newtonian and longitudinal gravitational potentials can be interpreted as a test of the strength of gravity (α) and the amount of curvature produced per unit rest mass (γ). Hence, we introduce a new parameter, ϖ , implicitly defined by the equation

$$\psi = (1 + \varpi)\phi, \quad (4)$$

which describes the departure from GR in the cosmological context. In terms of PPN parameters, $\varpi = \frac{\alpha}{\gamma} - 1$ which translates to $\varpi \approx 1 - \gamma$ for a weak departure from GR. Reexpressing the results of Ref. [22] on the comparison between lensing and dynamical measurements of galaxy masses, then $\varpi = 0.02 \pm 0.07$ (68% CL) on tens of kiloparsec scales. The same analysis can be extended

to a few hundred kiloparsec scales by making use of both strong and weak lensing measurements towards galaxy clusters combined with dynamical mass measurements of same clusters. Using the comparison between weak lensing and X-ray based masses in Ref. [24], we find $\varpi = 0.03 \pm 0.10$ (68% CL). At megaparsec scales corresponding to cosmological observations, unfortunately, there are no useful limits placed on ϖ .

A difference in the two gravitational potentials can also occur in GR when the gravitating source has anisotropic stress or shear. Specifically, the off-diagonal components of the perturbed Einstein equation yield

$$k^2(\phi - \psi) = 12\pi G a^2(\rho + p)\sigma, \quad (5)$$

where ρ , p are the energy density and pressure of the fluid component giving rise to the shear. Of course, there are other sources of shear, due to the contribution of relativistic species such as photons or neutrinos. Including the effects due to a departure from GR we obtain

$$k^2(\phi - \psi) = 12\pi G a^2(\rho + p)\sigma|_{\gamma,\nu} - \varpi k^2\phi. \quad (6)$$

Note that the difference between the two potentials, as well as individual shear components, are gauge-invariant. In the synchronous gauge, the potentials are

$$\phi = \eta - \mathcal{H}\alpha, \quad \psi = \dot{\alpha} + \mathcal{H}\alpha, \quad \alpha = \frac{1}{2k^2}(\dot{h} + 6\dot{\eta}) \quad (7)$$

where the dot indicates a derivative with respect to conformal time. Consequently, the shear parametrization equation may be recast in terms of the evolution equation for α :

$$\dot{\alpha} + (2 + \varpi)\mathcal{H}\alpha - (1 + \varpi)\eta = -12\pi G a^2(\rho + p)\sigma|_{\gamma,\nu}. \quad (8)$$

Hence, the static PPN-like relation between potentials in the conformal-Newtonian gauge becomes dynamical in the synchronous gauge. In the concordance cosmological model, which includes Einstein's cosmological constant within GR, photon and neutrino shear are negligible in the late universe. Hence, an observed difference between the potentials signals some non-standard behavior.

This behavior may be seen in gravitational phenomena such as the deflection of light. The deflection angle experienced by a beam of light moving through a potential ϕ now takes the form $\delta\theta = -2(1 + \varpi)\phi$. In the cosmological setting, we can find the lensing potential for the deflection of photons by adapting the results of Acquaviva *et al* [25], which amounts to replacing their Ξ with $\varpi\phi$, to obtain

$$\varphi(\hat{n}) = \int_0^{\chi_\infty} d\chi g'(\chi) [(2 + \varpi)\phi(\hat{n}, \chi)]. \quad (9)$$

Here χ is the comoving distance from the observer, $g'(\chi) = \chi \int_\chi^{\chi_\infty} d\chi' (1 - \chi/\chi') n(\chi')$, and $n(\chi)$ gives the distribution of background sources. This means the pattern of lensing of background galaxies, CMB, or any field of photons, will be influenced by $\varpi \neq 0$.

Similarly, the integrated Sachs-Wolfe effect (ISW) is affected by ϖ :

$$\frac{\delta T}{T}(\hat{n})|_{ISW} = \int d\chi ((2 + \varpi)\phi)_{,\tau}(\hat{n}, \chi). \quad (10)$$

Hence, combined measurements of the CMB with probes of large-scale structure, which are sensitive to ϕ through the Poisson equation, can be used to isolate the effects of ϖ .

Our focus on the *gravitational slip* between the potentials is motivated in part by the recent work of Bertschinger [26], as well as numerous investigations of scalar-tensor and generalized gravitational theories (*e.g.* [25, 27, 28, 29, 30]) and dark energy with anisotropic stress [31, 32]. Our intention is to use these existing tools and techniques, not to explore exotic theories and models, but to examine simple departures from GR with a cosmological constant.

III. COSMIC MODEL

It is evident that ϖ can be used to parametrize departures from Einstein's GR. A specific prediction for the gravitational slip in the cosmological context is obtained for a given theory. For example, the difference between the two conformal-Newtonian gauge potentials is $\phi - \psi = \delta(\ln L_{,R})$ in a theory of gravity described by a generalized Lagrangian $L(R, \varphi)$ [25], where R is the Ricci scalar curvature and φ is the gravity-coupled scalar field. The relation between a fourth-order gravitational theory and PPN parameters γ, β is given in Ref. [33]. The gravitational slip in a scalar-tensor theory including a scalar coupling to the Gauss-Bonnet Lagrangian is analyzed in Ref. [34]. In the Dvali-Gabadaze-Porrati (DGP) brane-world model of gravity [35, 36], recent studies of structure formation [37] derive a gravitational slip $\phi - \psi = 8\pi G a^2 \rho (\delta_m + 3\mathcal{H}\mathcal{V}_m/k^2)/\beta k^2$. Here β is a dimensionless coefficient which relates the five-dimensional cross-over scale, r_c , to the present-day Hubble constant, whereby $\varpi \approx \mathcal{O}(1/\beta)$. More examples can certainly be found. We would like to find a phenomenological description which captures the basic effects. The advantage we see is that we can streamline the search for non-GR behavior; if we find evidence for $\varpi \neq 0$ then we can focus on particular theories.

In general, we expect dark energy effects and the corresponding gravitational slip to turn on at late times. Hence, we propose a simple extension of the concordance, Λ CDM cosmological model. 1) We assume that the dark energy phenomena resembles a cosmological constant such that the background evolution of the expansion scale factor is driven by a component with equation-of-state $w = -1$. This maintains consistency with all observational constraints based on classical distances. 2) We assume an evolution for the gravitational slip which

grows large as the role of dark energy grows:

$$\varpi = \varpi_0 \frac{\rho_{DE}}{\rho_M} = \varpi_0 \frac{\Omega_{DE}}{\Omega_M} (1+z)^{-3}. \quad (11)$$

Our immediate goal is to constrain ϖ_0 . Based on our estimate of the local ratio between dark energy and matter (2), we expect $|\varpi_0| \lesssim 10$ in order to be consistent with the solar system limits on γ [19]. Next, using the approximation that $\rho_{DE}/\rho_M \sim 10^{-4}$ on the scale of a galaxy halo, then the limits of Refs. [22, 24] give $|\varpi_0| \lesssim 100$. The cosmological ratio, however, only grows significant at late times as the inferred dark energy comes to dominate. The source of ϖ , the slip between ϕ and ψ , may be interpreted as due to a departure from Einstein gravity or, alternatively, due to a dark energy component with anisotropic shear. 3) We employ a subset of Einstein's equations to evolve linear perturbations under the presumed non-GR theory, to be defined shortly. Hence, we are making the implicit assumption that a portion of the new field equations are equivalent to GR with an additional shear.

We pause to emphasize the distinction between dark energy with shear and departures from GR. Although the slip between the gravitational potentials can arise from either source, non-GR theories of gravity are not generally equivalent to GR and dark energy with shear. Our implementation of ϖ represents a only portion of the non-standard behavior resulting from different gravitational theories. Since observations already tightly constrain gross departures from GR, we do not expect to be missing much with our parametrization. To keep the phenomenology simple, we restrict attention to $w = -1$. (As well, dark energy or modified gravity with $w \neq -1$ would surely be detected through classical tests of cos-

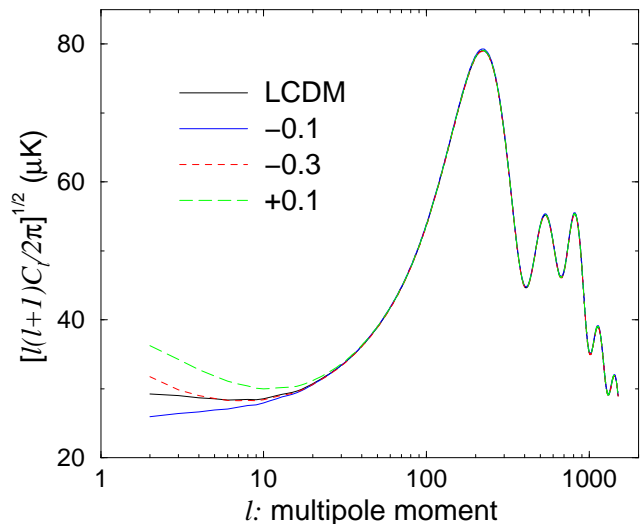


FIG. 1: The CMB anisotropy spectra under recipe R1 are shown for different values of ϖ_0 . R3 produces identical spectra. The backgrounds for all models are identical: $\Omega_m = 0.35$, $h = 0.65$ and dark energy with $w = -1$.

mology before the effects of shear are manifest.)

Different theories of gravitation predict different relations between stress-energy and spacetime curvature. Although we presume a Λ -like background evolution, the equations guiding cosmological perturbations must differ. In the absence of a specific theory of gravity, we intend to use the following recipe (R1): 3.i) Evolve radiation and matter fluid sources using conservation of the individual stress-energy tensors, as in the standard case; 3.ii) Evolve ϖ according to (11); 3.iii) Enforce the slip between ϕ and ψ by evolving α according to (8); 3.iv) Evolve the time-space perturbed Einstein equation $k^2\dot{\eta} = 4\pi Ga^2(\rho + p)\theta$ where θ is due to the radiation and matter sources only, not dark energy.

Recipe R1 describes a theory of gravity in which the time-space equation is unchanged, but the gravitational slip between ϕ and ψ evolves as given by ϖ . Alternatively, this describes a dark energy with shear perturbations but no fluctuations of the momentum density. Implicitly, there must be energy density and pressure perturbations of the dark energy. (Note that CMBfast [39] enforces energy conservation by applying the time-time perturbed Einstein equation, $\dot{h} = 2(k^2\eta + 4\pi Ga^2\delta\rho)/\mathcal{H}$, at each step. Instead, we use the definition of α to get \dot{h} .)

We have also considered alternate recipes. For recipe R2, the time-time perturbed Einstein equation is evolved in place of the time-space equation. This corresponds to a theory of gravity in which the time-time equation is unchanged; alternatively, the dark energy has no energy density fluctuations, but does have momentum density and pressure perturbations. The derivative $\dot{\eta}$ is obtained from the definition of α . Finally, for recipe R3, we drop the evolution of α and let the synchronous gauge variables h and η evolve as in the standard case. However, for matters of calculating the CMB anisotropies we convert to the conformal-Newtonian gauge with $\phi = \eta - \mathcal{H}\alpha$ but then artificially set $\psi = (1 + \varpi)\phi$.

We have modified a version of CMBfast [39] to evolve perturbations according to these schemes. The resulting CMB anisotropy spectra are shown in the following figures. All the models are spatially flat with $\Omega_m = 0.35$, $h = 0.65$, and normalized to have the same small angular-scale power spectra as the corresponding Λ CDM model. In Figure 1 we show the anisotropy power spectrum for different values of ϖ_0 under recipe R1. The alternative model, R3, produces identical spectra. Empirically, we see that for small, negative values of ϖ_0 close to -0.1 there is a cancellation between the ISW and SW effects which leads to a slight reduction in the amplitude of the lowest multipole moments. For these cosmological parameters, the minimum value of the quadrupole is achieved with $\varpi_0 \approx -0.15$. For large values of $|\varpi_0 + 0.15|$, the reduction turns into an enhancement. In Figure 2 we show the anisotropy power spectrum with $\varpi_0 = -0.1$ for different recipes. All three recipes produce similar results.

The growth of structure is also modified by the gravita-

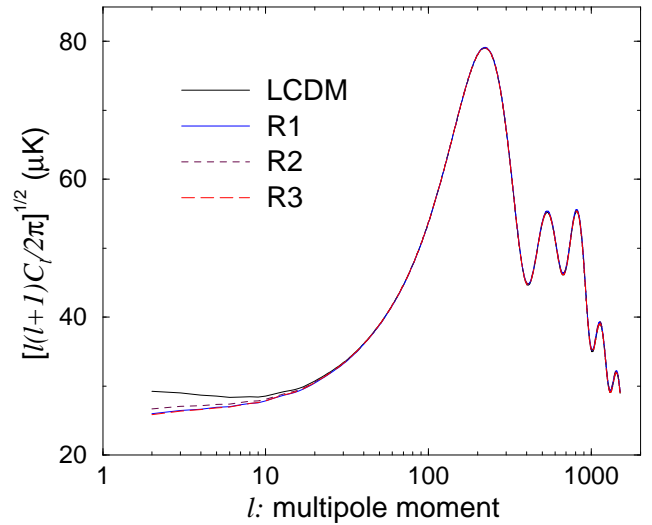


FIG. 2: The CMB anisotropy spectra under different recipes are shown for $\varpi_0 = -0.1$.

tional slip. According to recipe R1, the cold dark matter density contrast δ_c evolves as

$$\ddot{\delta}_c + \mathcal{H}\dot{\delta}_c = k^2(1 + \varpi)(\mathcal{H}\alpha - \eta). \quad (12)$$

where we have used $\dot{\delta}_c = -\dot{h}/2$, equation (8), and we have assumed that the shear due to radiation and neutrinos is negligible, as occurs beginning late in the matter-dominated era. Recipe R1 maintains the validity of the time-space perturbed Einstein equation, whereas the time-time equation is implicitly modified. If the modifications of the time-time equation are absorbed into a dark energy density perturbation, whereby $k^2\eta - \mathcal{H}\dot{h}/2 = -4\pi Ga^2(\delta\rho + \delta\rho_{DE})$, then the CDM density contrast evolution equation becomes

$$\ddot{\delta}_c + \mathcal{H}\dot{\delta}_c = 4\pi Ga^2(\delta\rho + \delta\rho_{DE})(1 + \varpi). \quad (13)$$

The effective dark energy density perturbation on the right-hand side is familiar from studies of quintessence dark energy. The novel feature of this equation is the $1 + \varpi$ factor which can lead to faster (slower) growth for $\varpi > 0$ (< 0).

The modification of standard cosmological perturbation theory to incorporate the effects of gravitational slip is not unique, as described above. There may be recipes other than R1-3 to include ϖ due to a departure from GR. Any particular non-GR theory makes a specific prediction for the behavior of ϖ as well as other effects. In the appendix, we study the behavior of a dark energy component with $w = -1$ which also generates a gravitational slip. However, as we have shown, the differences in the three recipes are small when ϖ_0 is small. Thus, despite the degeneracy, we find a practical advantage in this simple modeling of departures from GR.

IV. CMB ANISOTROPY SPECTRUM

Let us now investigate to what extent ϖ_0 can be constrained from current CMB observations. As shown in the previous section, a change in ϖ_0 will mostly affect the large angular scale CMB anisotropies through the ISW effect, leaving the acoustic peak structure unchanged. For a first study, we restrict the analysis to a flat, “concordance,” Λ CDM model with $\Omega_\Lambda = 0.76$, $\Omega_b = 0.02$, $h = 0.73$. We also make the choice of adiabatic and scalar inflationary perturbations with a spectral index of $n_s = 0.958$ and we fix the optical depth to $\tau = 0.08$ as suggested by Wilkinson Microwave Anisotropy Probe (WMAP) 3-year anisotropy and polarization measurements. We then let ϖ_0 vary and compare the CMB power spectra with the WMAP three year temperature anisotropy data [40, 41, 42, 43] (WMAP3). The likelihood is determined using the October 2006 version of the WMAP likelihood code available at the LAMBDA website [44]. In Figure 3 we plot the likelihood for ϖ_0 (under recipe R1) from WMAP. As we can see, the likelihood peaks at $\varpi_0 \sim -0.1$ although $\varpi_0 = 0$ is consistent with the data: the 95% CL range is $-0.28 < \varpi_0 < 0.05$. The preference for negative values of ϖ_0 is a direct consequence of the low measured CMB quadrupole. However, as shown in the previous section, very large negative values such that $|\varpi_0| \gtrsim 0.5$ enhance the quadrupole, making the model dramatically incompatible with the data.

It is also interesting to investigate possible variations of ϖ_0 by relaxing the assumption of $w = -1$ dark energy component. Let us indeed substitute the cosmological constant with a perturbed fluid with constant equation of state $p = w\rho$ and sound speed c_s^2 . An equation of state $w \neq -1$ and perturbations in the fluid would also change the large angular scale anisotropy. As it is

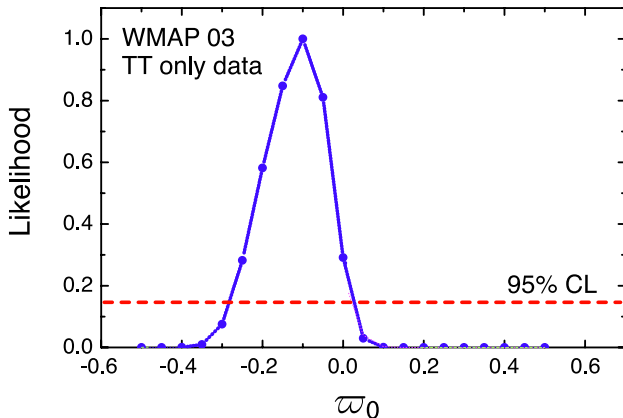


FIG. 3: Likelihood distribution for ϖ_0 due to WMAP 3-year CMB temperature measurements for a flat, “concordance,” Λ CDM model with $\Omega_\Lambda = 0.76$, $\Omega_b = 0.02$, $h = 0.73$, adiabatic and scalar inflationary perturbations with a spectral index of $n_s = 0.958$, and an optical depth of $\tau = 0.08$. The 95% CL range is $-0.28 < \varpi_0 < 0.05$ with a peak at a slightly negative value of ϖ_0 owing to the decrease in large-scale power.

well known (see e.g. [45, 46]) a geometrical degeneracy makes virtually impossible any determination of w from the position of the acoustic peaks in the CMB anisotropy spectrum. However, combined measurements constrain the equation of state parameter to a conservative range $-1.2 < w < -0.8$ (see e.g. [40]). In Figure 3 we show the likelihood for ϖ_0 when a dark energy fluid with equation of state $w = -0.8$ is considered. The other parameters have been chosen to reproduce the acoustic peak structure of the concordance model through the geometrical degeneracy relation. We therefore fix $\Omega_m = 0.298$, $h = 0.655$ and $\Omega_b = 0.0495$. We consider two possibilities for the sound speed parameter $c_s^2 = 1, 0$. As we can see, the likelihood for ϖ_0 is only moderately affected: we obtain $-0.38 < \varpi_0 < 0.05$ for $c_s^2 = 1$ and $-0.35 < \varpi_0 < 0.09$ for $c_s^2 = 0$ at 95% CL. We can therefore claim that current observations provide a conservative bound of $-0.4 < \varpi_0 < 0.1$ at 95% CL. This constraint is stronger than the PPN, lensing, and x-ray constraints on ϖ_0 , based on our model (11), because $\rho_{DM} \ll \rho_M$ on megaparsec scales and below.

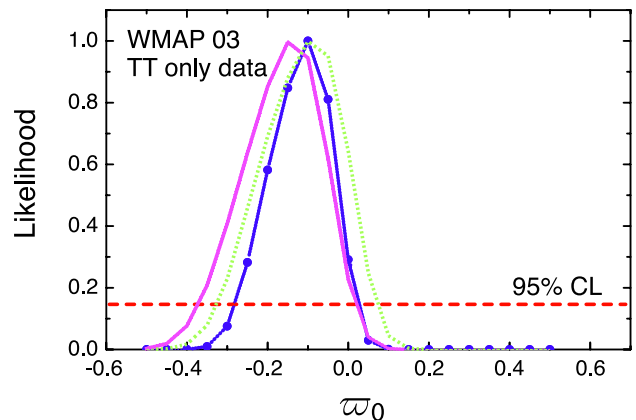


FIG. 4: Likelihood distributions for ϖ_0 due to WMAP 3-year CMB temperature measurements within the framework of different dark energy models. The curve with dots is for Λ CDM, as in the previous figure. The solid and dotted curves are for a dark energy model with $w = -0.8$, with parameters chosen to reproduce the acoustic peak structure of the Λ CDM model, but with dark energy perturbation sound speed $c_s^2 = 1, 0$. The sound speed does not have a strong effect on the likelihood range. Allowing for a range in equation of state $-1.2 < w < -0.8$, then we obtain a conservative bound $-0.4 < \varpi_0 < 0.1$ at 95% CL.

V. EXPECTATIONS FOR ISW AND WEAK LENSING MEASUREMENTS

Post-GR gravitational slip would also leave an imprint on the cross-correlation between the CMB and large-scale structure. We define the two-point angular cross-correlation between the temperature ISW anisotropy and

the dark matter fluctuation as (see e.g. [47]):

$$C^X(\theta) = \langle \Delta_{ISW}(\hat{\gamma}_1) \delta_{LSS}(\hat{\gamma}_2) \rangle, \quad (14)$$

where the angular brackets denote the average over the ensemble and $\theta = |\hat{\gamma}_1 - \hat{\gamma}_2|$. For computational purposes it is convenient to decompose $C^X(\theta)$ into a Legendre series such that,

$$C^X(\theta) = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l^X P_l(\cos(\theta)), \quad (15)$$

where $P_l(\cos \theta)$ are the Legendre polynomials and C_l^X is the cross-correlation power spectrum given by [48, 49]:

$$C_l^X = 4\pi \frac{9}{25} \int \frac{dk}{k} \Delta_{\mathcal{R}}^2 I_l^{ISW}(k) I_l^{LSS}(k), \quad (16)$$

where $\Delta_{\mathcal{R}}^2$ is the primordial power spectrum. The integrand functions $I_l^{ISW}(k)$ and $I_l^{LSS}(k)$ are defined respectively as:

$$I_l^{ISW}(k) = - \int e^{-\kappa(z)} \frac{d((2+\varpi)\phi_k)}{dz} j_l[kr(z)] dz \quad (17)$$

$$I_l^{LSS}(k) = b \int \Phi(z) \delta_m^k(z) j_l[kr(z)] dz, \quad (18)$$

where ϕ_k and δ_m^k are the Fourier components of the gravitational potential and matter perturbation, respectively; Φ is the galaxy survey selection function; $j_l[kr(z)]$ are the spherical Bessel functions; $r(z)$ is the comoving distance at redshift z and $\kappa(\tau) = \int_{\tau}^{\tau_0} \dot{\kappa}(\tau) d\tau$ is the total optical depth at time τ .

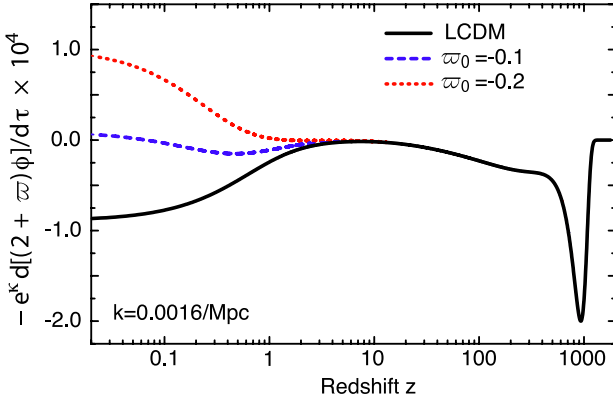


FIG. 5: Contributions to the integrated Sachs-Wolfe effect as a function of the redshift z for different values of ϖ_0 .

A change in ϖ could therefore change not only the value of I_l^{ISW} but also its sign respect to I_l^{LSS} . In Figure 5, we plot the values of the integrand $e^{-\kappa} d((2+\varpi)\phi_k)/d\tau$ at a scale of $k \sim 0.0016 \text{ Mpc}^{-1}$ as a function of different values of ϖ_0 . As we can see, while the early ISW at $z \sim 1000$ is left unchanged, a value of $\varpi_0 \sim -0.2$ produces a change in the sign of the late ISW contribution. This could indeed produce a negative cross-correlation with galaxy surveys.

Direct measurements of the cross-power spectrum C_l^X are more robust for likelihood parameter estimation since these data would be less correlated than measurements of $C^X(\theta)$. We therefore compute the cross-power spectrum C_l^X for different values of ϖ_0 assuming a galaxy survey with a selection function as

$$\Phi(z) \sim z^2 \exp[-(z/\bar{z})^{1.5}] \quad (19)$$

where \bar{z} , the median redshift of the survey, is $\bar{z} = 0.25$. In the past years, the WMAP temperature anisotropy maps have been cross-correlated with several surveys of Large Scale Structure (LSS) distributions and a positive correlation signal has been detected [50, 51, 52, 53, 54, 55]. As we can see from Figure 6, values of $\varpi_0 < -0.2$ would result in anti-correlation and a negative angular spectrum which is in disagreement with current observations at the $\sim 2\sigma$ level.

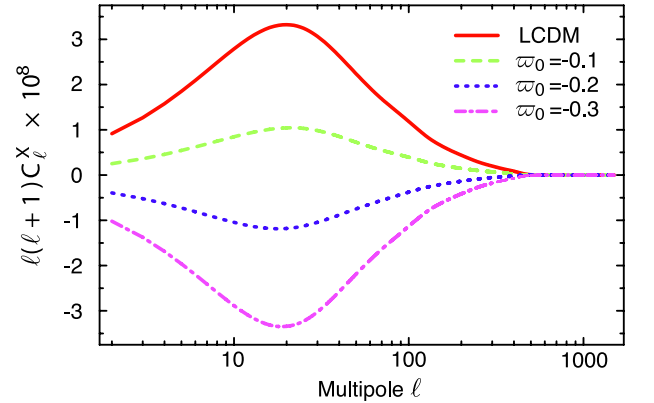


FIG. 6: Cross-correlation angular power spectrum between CMB temperature (ISW) and galaxy distribution as a function of ϖ_0 . A value of $\varpi_0 < -0.2$ changes the sign of the ISW, resulting in a negative cross-correlation, in conflict with observations.

In addition to the ISW effect, weak lensing measurements of the large-scale structure capture modifications imposed by ϖ . Using equation (9), we can write the angular power spectrum of lensing convergence κ as

$$C_l^\kappa = \frac{2}{\pi} \int k^2 dk [I^\kappa(k)]^2 P_{\phi\phi}(k), \quad (20)$$

where

$$I^\kappa(k) = \int d\chi g(\chi) (2+\varpi) j_l(k\chi). \quad (21)$$

In Fig. 7, we show the lensing convergence for our standard LCDM model and a model in which $\varpi_0 = -0.1$ and $\varpi_0 = 0.1$ in Eq. (11) assuming that all sources are at a fixed redshift of $z_s = 1$. As shown, there is an overall change in the amplitude of lensing convergence fluctuations such that when $\varpi_0 = -0.1$ (0.1) the fluctuation power spectrum is lower (higher) by about 8%. To see the modification associated with ϖ , the overall normalization

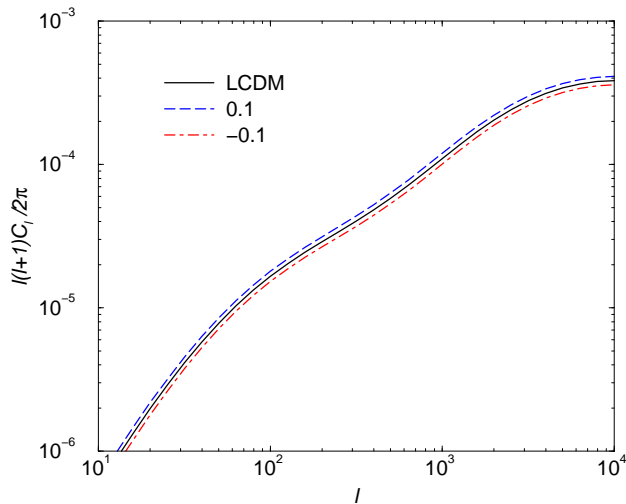


FIG. 7: Weak lensing convergence power spectrum for LCDM and for several values of ϖ_0 assuming that all sources are at $z_s = 1$.

of the convergence power spectrum, such as through σ_8 , must be known to an accuracy better than 8%.

Detection of modifications from ϖ with lensing statistics alone is impossible in the same manner that lensing measurements of individual foreground objects cannot be used to separate ϕ from ψ as the measurements are sensitive to the combination only. To constrain our post-GR parameter, as in Ref. [22], one must combine lensing measurements with an independent measurement which is sensitive to a different combination of the two parameters or to just one of the parameters. In the case of galaxy lensing, the comparison is made to dynamical mass estimates which are sensitive to ϕ . In this context, in the case of large-scale structure weak lensing, the weak lensing statistics must be compared with an additional probe of large-scale structure, such as the potentials probed by the galaxy distribution. While we have not carried out such an analysis here due to the as-yet large uncertainties in weak lensing measurements, in the future it may be possible to further constrain ϖ_0 through a combination of lensing measurements and galaxy clustering power spectrum.

VI. CONCLUSIONS

We have introduced a new cosmological variable to characterize the degree of departure from Einstein’s General Relativity. The new parameter, ϖ , is the cosmological analog of the parametrized post-Newtonian variable γ , which measures the amount of spacetime curvature per unit mass. Our study was motivated by various discussions in the literature on uses of large-scale structure measurements to establish departures from GR on scales beyond the solar system (e.g., [26]). In the cosmological context, the parameter ϖ measures the difference

between the Newtonian and longitudinal potentials in response to the same matter sources, as occurs in certain scalar-tensor and other modified theories of gravity. Equivalently, ϖ measures the scalar shear fluctuation in a dark energy component. In the context of a “vanilla” LCDM background cosmology, a non-zero ϖ signals a departure from GR or a fluctuating cosmological constant.

We have parameterized the time-evolution of ϖ through a simple phenomenological model in which $\varpi = \varpi_0(1+z)^{-3}\Omega_{\text{DE}}/\Omega_{\text{M}}$. The limit due to the observed cosmic microwave background temperature anisotropies from the WMAP3 data [40, 41, 42, 43] is $-0.4 < \varpi_0 < 0.1$ at the 95% confidence level. The detection of a positive ISW effect [50, 51, 52, 53, 54, 55] further limits $\varpi_0 > -0.2$. Within the context of our model for $\varpi(z)$, these constraints are tighter than the solar system time-delay and galaxy-scale gravitational lensing tests of GR by an order of magnitude.

We have considered possibilities to improve these constraints and have discussed the potential use of both ISW and weak lensing measurements of the large-scale structure. In both cases, these measurements must be combined with additional probes of large-scale structure, such as the galaxy power spectrum, to break degeneracies between ϖ and parameters. While existing data are limited given the large uncertainties involved, weak lensing measurements combined with galaxy clustering may be an ideal probe of ϖ . Further evidence that $|\varpi| \ll 1$, or a detection of $\varpi \neq 0$, will provide useful clues as to the validity of GR and the nature of dark energy.

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APPENDIX A: W=-1 DARK ENERGY FLUCTUATIONS

It is an apparent contradiction that a cosmological constant can fluctuate. Einstein’s cosmological term is regarded as a universal constant, leading to a stress-energy $T_\nu^\mu = -\Lambda g_\nu^\mu/8\pi G$. The fluctuations must vanish by definition, $\delta T_\nu^\mu = 0$. To avoid confusion, we will keep the cosmological constant pristine – anything otherwise would be a misnomer. However, if the dark energy is a constant only after some suitable averaging or expectation value is obtained, then it is not clear that its fluctuations must also vanish. An example which has been explored recently is the ghost condensate [56], consisting of an effective field theory which gives rise to a cosmological fluid with equation-of-state $w = -1$, but also supports scalar fluctuations and a novel dispersion relation. The full implications of such a theory are beyond the scope

of our investigation, but serves as a motivating proof-of-principle.

In the absence of a specific model, we describe the dark energy as an imperfect fluid with equation-of-state $w = p/\rho$ for the homogeneous, isotropic background, but which also supports anisotropic stress or shear, σ . Examples of dark energy models with anisotropic stress are given in Refs. [31, 32, 57, 58]. The equations of motion for the linearized perturbations are given by

$$\begin{aligned}\dot{\theta} &= -\mathcal{H}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta p/\delta\rho}{1+w}k^2\delta - k^2\sigma + k^2\psi \\ \dot{\delta} &= -(1+w)(\theta - 3\dot{\phi}) - 3\mathcal{H}\left(\frac{\delta p}{\delta\rho} - w\right)\delta\end{aligned}\quad (\text{A1})$$

which are merely equations (30a, b) of [23]. Note that we neglect vector and tensor perturbations, which may play a role in dark energy models with shear. The pressure and energy density perturbations are linked by $c_s^2 \equiv \delta p/\delta\rho$. If we consider that $\mathcal{V} = (1+w)\theta$ is the relevant variable to describe the momentum density perturbation, and $\Pi = (1+w)\sigma$ describes the shear, then the equations become

$$\begin{aligned}\dot{\mathcal{V}} &= -\mathcal{H}(1-3w)\mathcal{V} + c_s^2 k^2\delta - k^2\Pi + (1+w)k^2\psi \\ \dot{\delta} &= -\mathcal{V} + 3(1+w)\dot{\phi} - 3\mathcal{H}(c_s^2 - w)\delta.\end{aligned}\quad (\text{A2})$$

Next, we set $w = -1$ and make the reasonable assumption that the gravitational potentials ϕ and ψ remain finite but that \mathcal{V} , $\delta\rho$, δp , Π do not automatically vanish. The equations become

$$\begin{aligned}\dot{\mathcal{V}} &= -4\mathcal{H}\mathcal{V} + c_s^2 k^2\delta - k^2\Pi, \\ \dot{\delta} &= -\mathcal{V} - 3\mathcal{H}(c_s^2 + 1)\delta\end{aligned}\quad (\text{A3})$$

There is no longer any source term representing the inhomogeneities of the surrounding medium; the fluctuations only feel the background, cosmic expansion. Of course, these fluctuations still feed in to the metric perturbation equations. Note that the set of equations (A3) become trivial in the case of a scalar field, for which $\Pi = \delta = \mathcal{V} = 0$ in the limit $w \rightarrow -1$.

The system of equations (A3) can be expressed as a single, second order equation for δ :

$$\ddot{\delta} + (7 + 3c_s^2)\mathcal{H}\dot{\delta} + [(12\mathcal{H}^2 + 3\mathcal{H}')(1 + c_s^2) + c_s^2 k^2]\delta = k^2\Pi. \quad (\text{A4})$$

In the absence of shear, the short-wavelength modes are stable provided $c_s^2 \geq 0$. The long-wavelength modes decay as a^{-3} , $a^{-3(1+c_s^2)}$ in a radiation-dominated epoch, and as $a^{-7/2}$, $a^{-3(1+c_s^2)}$ under matter. Any such fluctuations are of little interest since the decay is so rapid.

A wider variety of behavior is possible with the addition of shear. The sound speed c_s no longer serves as the relevant quantity guiding the propagation and stability of

perturbations. Now, $c_s^2\delta \geq \Pi$ is required for stability, and the high-frequency phase velocity is $v = (c_s^2 - \Pi/\delta)^{1/2}$. This corresponds to a time and scale-dependent cosmic post-GR parameter

$$\varpi = -12\pi G a^2 \rho \Pi / k^2 \phi. \quad (\text{A5})$$

The properties of the shear, and therefore the cosmic post-GR parameters, are dictated by the details of any particular model. The quartic dispersion relation for the excitations in the ghost condensate can be modeled by

$$\Pi = -\frac{k^2}{M^2}\delta. \quad (\text{A6})$$

where M is the energy scale below which the effective field theory is valid. Stability of modes within the horizon requires $c_s^2 \geq 0$, in which case the perturbations decay rapidly. Next, a phenomenological evolution equation has been proposed by Hu [58], whereby

$$\dot{\Pi} + 3\mathcal{H}\Pi = \frac{8}{3}c_{\text{vis}}^2\theta \quad (\text{A7})$$

(when translated into our notation) in the conformal-Newtonian gauge. The viscosity sound speed c_{vis} modulates the source of the shear, which otherwise decays. However, a phenomenological model which reflects equation (2), whereby the shear-effects turn on as the local dark-energy density overtakes the matter, can be achieved if

$$\Pi \equiv \frac{1}{3}\varpi_0\delta. \quad (\text{A8})$$

In this case, $\varpi \approx \varpi_0\delta\rho_{DE}/\delta\rho_{TOT}$ on scales within the horizon, which resembles our model (11). We can also make a connection with Hu's formula by plugging (A8) into (A3), such that

$$\dot{\Pi} + 3\mathcal{H}(1 + c_s^2 - \frac{1}{3}\frac{d\ln\varpi_0}{d\ln a})\Pi = -\frac{1}{3}\varpi_0\mathcal{V}. \quad (\text{A9})$$

This corresponds to a viscosity sound speed $c_{\text{vis}}^2 = -\varpi_0(1+w)/8$, but a damping rate which is now variable. With $c_s^2 < -1$ and a constant or growing parameter ϖ_0 , the anisotropic scalar shear can actually grow. It is not clear what would be an appropriate choice of initial conditions for fluctuations in this dark energy component, since adiabaticity requires $\delta = \mathcal{V} = 0$. Initial conditions which are set by some measure of the relative contribution to curvature perturbations, $\delta R = -8\pi G(\delta\rho - 3\delta p)$, must be finely tuned in order to force a non-negligible gravitational slip at late times. In this investigation, however, we have focused directly on the slip rather than fluctuating dark energy models.

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